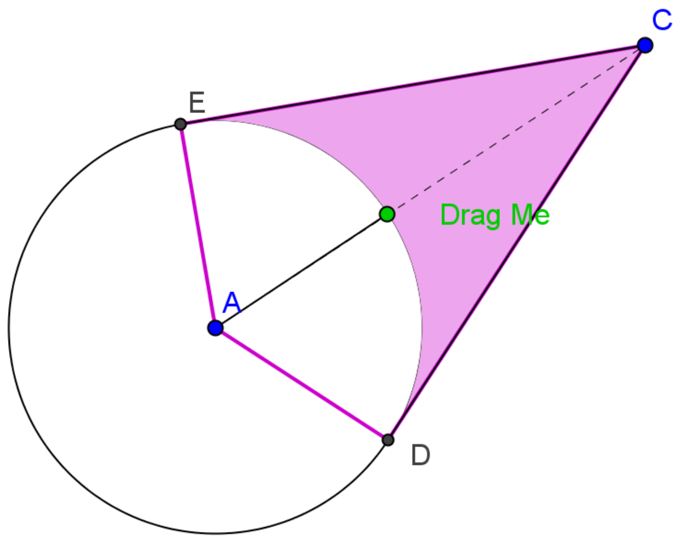

This is just ONE way to go about finding a relationship between the radius of the circle and the area of the shaded region.

Recall, AC is held constant.

I will find the area of triangle AEC, subtract the area of the sector from E to DRAG ME, then multiply by 2.

The area of triangle AEC is

$$\frac{1}{2}(AE)(EC)=\frac{1}{2}(r)\sqrt{(AC^2-r^2)}$$

The area of the sector is

$$\pi r^2\frac{\measuredangle EAC}{2\pi}=\frac{1}{2}r^2\measuredangle EAC$$

The measure of angle EAC is, in terms of the givens in the problem, is $\arccos(\frac{r}{AC})$. Therefore, the area is $\frac{1}{2}r^2\arccos(\frac{r}{AC})$.

The area of the shaded region is twice the difference between $\frac{1}{2}(r)\sqrt{(AC^2-r^2)}$ and $\frac{1}{2}r^2\arccos(\frac{r}{AC})$, which is

$$2\cdot\left(\frac{1}{2}(r)\sqrt{AC^2-r^2}-\frac{1}{2}r^2\arccos\left(\frac{r}{AC}\right)\right)$$

$$=r\cdot\sqrt{AC^2-r^2}-r^2\cdot\arccos\left(\frac{r}{AC}\right)$$